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PART B

Operator-algebraic geometry in the unit ball

OAGUB

Contents

B1 SCIENTIFIC AND TECHNOLOGICAL QUALITY

B1.1 Scientific and technological quality, including any interdisciplinary and multidisciplinary aspects of the proposal

B1.1.1 Definitions and notation

Let d be an integer, and let $\mathbb{B}_d$ denote the open unit ball in $\mathbb{C}^d$. The Drury–Arveson space, denoted H^2_d, is the space of all analytic functions f on $\mathbb{B}_d$ that have a Taylor series

$$f(z) = \sum_{\alpha \in \mathbb{N}^d} c_\alpha z^\alpha \tag{1}$$

such that

$$\sum_{\alpha \in \mathbb{N}^d} |c_\alpha|^2 \frac{\alpha_1! \cdots \alpha_d!}{(\alpha_1 + \ldots + \alpha_d)!} < \infty. \tag{2}$$

Defining $\|f\|^2$ to be equal to the left hand side of (2) for f as in (1) determines a Hilbert space structure on H^2_d.

For $i = 1, \ldots, d$, define $S_i : H^2_d \to H^2_d$ to be multiplication by the ith coordinate, i.e.,

$$[S_i f](z) := z_i f(z) \ , \ f \in H^2_d.$$

The d-tuple $S = (S_1, \ldots, S_d)$ is called the d-shift.

The space H^2_d is also a reproducing kernel Hilbert space (RKHS) with reproducing kernel

$$k_w(z) = \frac{1}{1 - \langle z, w \rangle}.$$

Let $\mathscr{M}_d$ denote the multiplier algebra of H^2_d:

$$\mathscr{M}_d = \{f : \mathbb{B}_d \to \mathbb{C} \big| fh \in H^2_d, \text{ for all } h \in H^2_d\}.$$

The algebra $\mathscr{M}_d$ is the weak-operator closed algebra generated by S. We define $\mathscr{A}_d$ to be the norm closed algebra generated S.

Let $\mathbb{C}[z]$ denote the algebra of complex polynomials in d variables, and let $I \triangleleft \mathbb{C}[z]$ be a radical homogeneous ideal. Let $Z(I)$ denote the zero locus of the ideal I, and put $V = Z(I) \cap \mathbb{B}_d$ and $\partial V = Z(I) \cap \partial \mathbb{B}_d$. The algebra of restrictions to V

$$\mathscr{A}_V := \{f\big|_V : f \in \mathscr{A}_d\}$$

is completely isometrically isomorphic to the universal operator algebra generated by a row contraction subject to the relations in I [62, 71]. The elements of the algebra $\mathscr{A}_V$ act as multipliers on the RKHS

$$\mathscr{F}_V := \overline{\operatorname{span}}\{k_w : w \in V\}.$$

Further, if we denote by $\mathscr{M}_V$ the weak-operator closure of $\mathscr{A}_V$, then we have $\mathscr{M}_V = \mathscr{M}_d\big|_V$, and $\mathscr{M}_V$ is the multiplier algebra of $\mathscr{F}_V$ [30].

B1.1.2 Objectives and problems

In this project, I plan to study the interaction between operator theory, function theory and algebraic geometry in the spaces $\mathscr{F}_V$. In particular, I am interested in using function theoretic and algebraic geometric methods to obtain asymptotic norm estimates on operators on the spaces $\mathscr{F}_V$. The main problems I intend to address are as follows.

Problem B1.1 (*Essential norm problem*) Compute the essential norm of elements in $\mathscr{A}_V$. This is equivalent to determining the operator algebraic structure of image $\mathscr{A}_V/\mathscr{K}$ of $\mathscr{A}_V$ in the Calkin algebra $B(\mathscr{F}_V)/\mathscr{K}$.

Problem B1.2 (*C*-envelope problem*) Describe the C*-envelope of $\mathscr{A}_V/\mathscr{K}$.

Problem B1.3 (*Stable division problem*) Prove an effective version of *Hilbert's Basis Theorem*. This can mean several things, here is one example (the clearest one): Prove that for every homogeneous ideal I there exists a generating set $\{f_1,\dots,f_k\}$ for I, which has the following property: *There is a constant C such that for all $h \in I$, there are $g_1,\dots,g_k \in \mathbb{C}[z]$ such that*

$$h = \sum g_i f_i$$

and

$$\sum \|g_i f_i\| \leq C\|h\|.$$

Problem B1.4 (*Arveson's conjecture*) Prove that the tuple of operators $Z = (Z_1,\dots,Z_d)$ obtained by compressing the d-shift to $\mathscr{F}_V$ is essentially normal, meaning that for all i,j, the commutator

$$[Z_i, Z_j^*] = Z_i Z_j^* - Z_j^* Z_i$$

is compact. A much more ambitious version of this problem is the following: *to prove that for all i,j, the operator $|[Z_i,Z_j^*]|^p$ is trace class, for all $p > \dim V$.* This implies that $[Z_i,Z_j^*]$ is compact, but is much stronger.

B1.1.3 State-of-the-art

Since the appearance of Arveson's influential paper [10], the Drury–Arveson space H^2_d (which was essentially introduced two decades earlier by S. Drury [38]), is one of the most central objects of study in the theory of *Hilbert modules* [35, 22].

Problem B1.4 is a special case of a notorious conjecture due to Arveson [12]. A solution of any one of the first three problems will put us in a better position to attack the fourth problem.

Arveson's original conjecture is more general than Problem B1.4, in that it deals with arbitrary quotients of $H^2_d \otimes \mathbb{C}^r$ by graded submodules. Many works were devoted to this conjecture, but only partial results have been obtained thus far. It is known that the conjecture is true for submodules generated by monomials [13, 33], for $d \leq 3$ and for principal modules [44]. To the best of my knowledge, the paper [40] contains all the cases for which the original conjecture holds, except those in [48]. Arveson showed that it is sufficient to prove the conjecture for submodules generated by vector–valued *linear* polynomials [14]. In [68] I reduced the problem further to *scalar*–valued, quadratic polynomials. The version I propose to study is already beyond the current state-of-the-art, and contains the most interesting instances for which a resolution of the conjecture is sought.

One may give an equivalent formulation of Problem B1.4 in terms of computation of the essential norm of elements in the C*-algebra $C^*(\mathscr{A}_V)$ generated by $\mathscr{A}_V$. Problem B1.1 sets the modest goal of merely computing the essential norm of elements in the much smaller algebra $\mathscr{A}_V$. In all cases where the answer is known, the essential norm $\|f\|_e$ of a function $f \in \mathscr{A}_V$ is given by $\sup_{z\in V}|f(z)|$ [13, 14, 33, 40, 44, 68].

The C*-envelope of an operator algebra was introduced in 1969 by Arveson [8, 9]. A decade later Hamana proved its existence [45], and since then for quite some time there were no techniques available by which one could compute the C*-envelope of an operator algebra or an operator system. However, during the last thirteen years, C*-envelopes of several classes of operator spaces and operator algebras have been computed [10, 16, 32, 47, 58, 60, 61]. Some of these techniques may be adapted to approach Problem B1.2. As above, in cases where Arveson's conjecture is known to hold true then Problem B1.2 is solved; it is known in those cases to be equal to $C(\partial V)$ [13, 14, 33, 40, 44, 68].

Problem B1.3 was raised by me quite recently in [68] as a new perspective on Arveson's conjecture. We say that an ideal I has the *stable division property* if it has a generating set $\{f_1, \dots, f_k\}$ as in Problem B1.3. Eschmeier considered a different line of attack on Arveson's conjecture [40], and showed that whenever an ideal I has the stable division property his approach works. Recently, Kennedy introduced a new approach to the essential normality problem, the key to this approach being the notion of an angle between subspaces [48]. Kennedy established close connections between his approach and the stable division property, and obtained some new results regarding the latter.

B1.1.4 Expected results

As mentioned above, Problem B1.4 is a conjecture that has already gained a little bit of a reputation. To be precise, a much more general version of the conjecture is believed to hold, and there is even a stronger conjecture due to Douglas [34] for which there is also plenty of evidence [33, 40, 44, 68]. I do not wish to take upon myself the assault of the conjecture in its full generality, but rather to concentrate on Problem B1.4, and I expect that, due to the extra analytic and geometrical tools which are available in this special setting, I will be able to make progress towards a solution. In any case, Problem B1.4 will serve as the guiding star of this project.

I have two alternative plans for attacking Arveson's conjecture. The first plan is tackle Problems B1.1 and B1.2 in tandem, and combine their solution to solve Problem B1.4. Loosely, I expect that the essential norm $\|f\|_e$ of a function $f \in \mathscr{A}_V$ is given by $\sup_{z\in V}|f(z)|$, and this would imply that $\mathscr{A}_V/\mathscr{K}$ can be identified with $A(\partial V)$, defined to be the norm closed subalgebra of $C(\partial V)$ generated by the polynomials. Further, I expect the C*-envelope of $\mathscr{A}_V/\mathscr{K}$ is equal to $C^*(\mathscr{A}_V)/\mathscr{K}$. On the other hand, the C*-envelope of $A(\partial V)$ is equal to $C(\partial V)$. Putting all this together will imply that $C^*(\mathscr{A}_V)/\mathscr{K} = C(\partial V)$, and this would show that Arveson's conjecture holds true in this case.

My alternate plan is to derive the validity of the conjecture from the existence of some kind of effective *Hilbert's Basis Theorem*, that is, by showing that some kind of polynomial division with bounds is possible. This is a novel approach to Arveson's conjecture which I introduced recently in [68] where the first results on the stable division problem were obtained. It is rather ambitious to expect to obtain exactly the version that was stated above in Problem B1.3. A very good beginning would be the following:

For every radical homogeneous ideal $I \triangleleft \mathbb{C}[z]$, to show that there exists a generating set $\{f_1, \dots, f_k\}$ and two sequences $\{C_n\}_{n=0}^\infty$ and $\{\varepsilon_n\}_{n=0}^\infty$, such that for all homogeneous $h \in I$ of degree n, there exist $g_1, \dots, g_k \in \mathbb{C}[z]$ such that

$$\|h - \sum g_i f_i\| \leq \varepsilon_n \|h\|$$

and

$$\sum \|g_i f_i\| \leq C_n \|h\|.$$

In unpublished preliminary work, I was able to show that if I has satisfies the conditions above where $\varepsilon_n \to 0$ and if C_n does not grow too fast, then I satisfies Arveson's conjecture. It is interesting to study what kind of bounds can be obtained in general.

In [68] I approached the problem of stable division using methods from computational algebraic geometry [24]. Discussions with researchers in computational algebraic geometry, as well as efforts to mine the literature [1, 18, 19, 24, 25, 49, 51, 59], convinced me that these results in computational algebraic geometry were not available off–the–shelf, and it seems that there is a whole fertile research area here waiting to be explored.

After some experience with the problem, I reached the opinion that the algorithmic methods from computational algebraic geometry might be too weak to solve the problem of stable division, and I seek to combine these methods with techniques from hard analysis. In some recent papers (e.g.

[23, 37]), methods from harmonic analysis and PDE were used to obtain norm control in related problems, and this seems like a promising route.

The problems I wish to investigate lie at the meeting point of several vital fields of mathematics: operator theory, functions of several complex variables, operator algebras, topology and algebraic geometry. The problems are of a pure mathematical nature, and are not studied with a particular application in mind. However, every one of the fields mentioned above has countless applications in science and engineering, and these applications were made possible only by the work of previous generations of mathematicians. I find it very likely that a success of this project will have a considerable impact on current mathematical research in these fields and therefore also have some (perhaps indirect) effect on the body of knowledge beyond mathematics.

The part of this project that is closest to applications is the part dealing with the stable division problem. Any advances made towards the solution of the stable division problem might lead to a better understanding of some algorithms of computational algebra [24], algorithms that are employed on a daily basis in numerous applied fields, from robotics [25] to oil drilling [50].

B1.2 Appropriateness of research methodology and approach

B1.2.1 Approach to Problem B1.1

The multiplier algebra $\mathscr{M}_d$ is known to be the universal multiplier algebra of a complete Pick kernel [7, 27, 57, 63, 2], and this is one of the reasons why it and its normed closed counter-part $\mathscr{A}_d$ have been at the focus of many studies in the last decade (see the monographs [3] and [65]). By a result of Arveson [10], the C*-algebraic quotient $C^*(\mathscr{A}_d)/\mathscr{K}$ is isomorphic to $C(\partial\mathbb{B}_d)$; hence the essential norm of any element $f \in \mathscr{A}_d$ is equal to $\sup_{z\in\partial\mathbb{B}_d}|f(z)|$. For elements in $\mathscr{M}_d$ this is no longer true [41].

The quotient algebras $\mathscr{A}_V$ and $\mathscr{M}_V$ have also received some attention [6, 30, 31, 62], but it seems that the problem of computation of the essential norm of elements in these algebras has been left largely untouched.

There is a technical result of Arveson [14] that can be used to show that essential norm of a polynomial is the same when considered as a multiplier on any one of the following spaces: the Drury–Arveson space, the Hardy space of the unit sphere, and the Bergman space of the unit ball. I plan to extend this result to the quotient spaces $\mathscr{F}_V$, transferring the problem to quotients of the Hardy or the Bergman space, and then to use the function theoretical description of the Hardy or the Bergman spaces to prove that the essential norm of an element $f \in \mathscr{A}_V$ is equal to $\sup_{z\in V}|f(z)|$.

The advantage of working in the Hardy or Bergman spaces will be that the norm in those spaces is given by an integral formula, and therefore it is more amenable to analytic techniques that are already available [72].

It should be noted that if Arveson's conjecture is true then the essential norm of an element $f \in \mathscr{A}_V$ *must* be equal to $\sup_{z\in V}|f(z)|$; thus it is expected that there will be no obstructions to completing this step.

B1.2.2 Approach to Problem B1.2

Among the several techniques that were developed to compute C*-envelopes [10, 16, 32, 47, 58, 60, 61], there is one I find most promising. Arveson's original approach to the C*-envelope was through *boundary representations* [8, 9]. These were only shown to exist recently [15], and it seems that there has not yet been sufficient opportunity to make use of this powerful tool. It is my hope that this would be the key to the problem.

There are two good reasons to expect that the C*-envelope of $\mathscr{A}_V/\mathscr{K}$ is equal to $C^*(\mathscr{A}_V)/\mathscr{K}$. The first is that this is what happens generally in the classical case of a function algebra inside a commutative C*-algebra. The second reason is that if Arveson's conjecture is true, then this *must* be true, and as I have already mentioned, there is plenty of evidence in favour of Arveson's conjecture.

B1.2.3 Approach to Problem B1.3

The sought after result — that every radical ideal has the stable division property — shares some resemblances to Corona theorems and Toeplitz–Corona theorems ([21, 23, 65]). In a break-through paper, S. Costea, E. Sawyer and B. Wick solved the Corona problem for multipliers on Drury–Arveson space (in fact they did more than that, see [23]). They relied on a Toeplitz-Corona theorem due to J. Ball, T. Trent and V. Vinnikov [17]. Their methods might be useful for this problem.

Let me briefly outline the connection. The Costea–Sawyer–Wick "Baby Corona Theorem" states that given functions $f_1,\ldots,f_k \in \mathscr{M}_d$ such that

$$\sum |f_i| \geq \delta \ \text{ on } \mathbb{B}_d, \tag{3}$$

and $h \in H^2_d$, there exist functions $g_1,\ldots,g_k \in H^2_d$ such that $\sum g_i f_i \equiv h$, and further, $\sum \|g_i\| \leq C(\delta)\|h\|$. In particular, $\sum \|g_i f_i\| \leq C\|h\|$. In the language of stable division, this shows that every basis for the trivial ideal $\mathbb{C}[z]$ is a "stable division" basis.

Now, if $\{f_1,\ldots,f_k\}$ is a generating set for a *nontrivial* ideal I, then (3) certainly does not hold, because the f_i's all vanish on $V = Z(I)$. But (3) does hold *away from* V. Further, we cannot solve $\sum g_i f_i \equiv h$ for h that does not vanish on V, but if we are only trying to solve for $h \in I$ we have a chance to succeed — in fact, algebraically we can succeed by definition. Thus we need to adapt the "Baby Corona Theorem" to the case where the f_i's possibly vanish on V, but in return we only have to solve $\sum g_i f_i \equiv h$ for functions h in I.

B1.2.4 Approach to Problem B1.4

As I indicated above, Problem B1.4 serves as the guiding star of this project, and I have outlined above two paths for approaching this problem: either by combining the solutions to problems B1.1 and B1.2, or by way of solving Problem B1.3.

In [68] I demonstrated how a solution of Problem B1.3 implies a solution to (the difficult version of) Problem B1.4. As for the first approach, the idea is as follows. If the essential norm $\|f\|_e$ of a function $f \in \mathscr{A}_V$ is given by $\sup_{z\in V}|f(z)|$, then $\mathscr{A}_V/\mathscr{K}$ can be identified with the norm closed subalgebra of $C(\partial V)$ (continuous functions on ∂V) generated by the polynomials; let us denote this norm closed algebra by $A(\partial V)$. It is not hard to see that the C*-envelope of $A(\partial V)$ is equal to $C(\partial V)$. Now, if the C*-envelope of $\mathscr{A}_V/\mathscr{K}$ is equal to $C^*(\mathscr{A}_V)/\mathscr{K}$, then we must have $C^*(\mathscr{A}_V)/\mathscr{K} = C(\partial V)$, whence the C*-algebra $C^*(\mathscr{A}_V)/\mathscr{K}$ is commutative. This implies that for all i,j, the commutator

$$[Z_i, Z_j^*] = Z_i Z_j^* - Z_j^* Z_i$$

is compact, which is the content of Problem B1.4.

It should be noted at this point, that the approach just described will only prove the weaker version of Problem B1.4. To solve the stronger version of Problem B1.4, I will have to solve Problem B1.3.

B1.3 Originality and innovative nature of the project, and relationship to the 'state of the art' of research in the field

In Section B1.1.3 I expanded on the 'state of the art'. In the following paragraphs I will treat the originality and innovative nature of the project, and the relationship to the 'state of the art' of research in the field.

There have been several shots made at Problem B1.4, and it seems that new ideas are needed. Both of the approaches I propose are novel. On the other hand, both of my approaches rely on deep and powerful mathematical theories, and there is an array of technical tools at my disposal.

The stable division property was introduced by me very recently in [68]. I believe that the problem of finding an Effective Basis Theorem is central to the study of Hilbert modules and goes beyond Arveson's conjecture; it is a fundamental problem in commutative multivariable operator theory. Any progress in this direction will spur much excitement and activity.

The idea to use the C*-envelope to prove Arveson's conjecture is completely new one.

All the evidence that has been gathered up to now supports Arveson's conjecture, therefore it seems that there should be no obstructions to solving problems B1.1, B1.2 and B1.3. However, if an obstruction is to be found for solving one of the first three problems it would be a triumph, since it would provide — for the first time — some idea as how to obtain a counter example to Arveson's conjecture.

B1.4 Timeliness and relevance of the project

I propose to study a set of problems that are at the pivotal junction where operator theory, function theory and algebraic geometry meet. The success of this project will shed new light on these fields, and reveal new links between them. The ultimate goal of this project is to solve one of the most stupefying conjectures in multivariate operator theory. Even partial progress will be a great success.

There will be a touch of irony if we succeed in proving Arveson's fourteen-year-old conjecture by applying Arveson's forty-year-old theory of C*-envelopes. Only recently did Arveson complete his arsenal of tools for analyzing C*-envelopes [15], and unfortunately he is no longer with us to show us how to use the powerful tools that he invented. We must continue this quest without him.

Let me elaborate on the significance of a solution to Problem B1.4. This conjecture has some profound consequences; see the discussions in [12, 13, 14, 33, 34, 44]. One of these consequences would be the stability of the curvature invariant of a commuting row contraction. The curvature invariant was introduced by Arveson [11] in his study of commuting row contractions. The paper [11] contained some striking results, but left open the question whether this invariant is stable under compact perturbations. In [12], Arveson connected the curvature invariant to multivariable Fredholm theory [26] and showed that if the conjecture holds, then the curvature invariant is stable under compact perturbations, similarity and homotopy.

Another consequence of Arveson's conjecture would be the explicit construction of elements of the odd K-homology groups of certain varieties. This aspect is discussed in [14, 33, 34, 44]. Here are some details. As in the description of Problem B1.4, let I be a homogeneous variety, and let Z be the compression of S to $I^\perp$. If $[Z_i, Z_j^*]$ is compact for all i, j, then we have the following exact sequence

$$0 \longrightarrow \mathscr{K} \longrightarrow C^*(Z) \longrightarrow C(\partial V). \longrightarrow 0 \qquad (4)$$

This extension of $\mathscr{K}$ by $C(\partial V)$ is the homology element mentioned above. Douglas conjectured that this element corresponds to the fundamental class determined by the almost complex structure on ∂V, and this has been verified in some cases.

It is my hope that the proposed research project will contribute to the European Mathematical Community in several ways, even before it is complete. First and foremost, I believe that other researchers will be attracted to the interesting and challenging problems that arise in this project, and that the problem of stable division will become a small research area of its own. Further, I think that the proposed project has great potential for collaborations between mathematicians from around the world and with different areas of research expertise. I already have collaborators in Canada, Europe and the USA who are interested in this circle of problems. Finally, the problems I will study are on the one hand challenging, while on the other hand quite concrete, and I anticipate that they will attract graduate students and postdoctoral fellows to work with me.

On a personal level, the CIG grant will help me in various aspects of my career: BGU's standard contract for new researchers is for a period of three years with an option to extend the contract for a further two years. The extending of the work contract and the granting of a permanent position at the end of five years will depend on my performance during the initial period. The integration grant will have a significant impact on my ability to build a high-quality academic track record during this period, as it will provide me with the resources necessary to recruit students and to establish a fruitful research group. The integration grant is also essential for cultivating international collaborations with researchers in the ERA and beyond through mutual visits. To qualify for an extension of the initial appointment and ultimately for a permanent position, I must be successful in all these activities. The grant will thus have a direct and significant effect on my career prospects.

B2 RESEARCHER

My main line of research lies in operator theory and in the theory of operator algebras. To be a little more specific, most of my work has been in dilation theory, noncommutative dynamics and E-semigroups, completely positive maps and semigroups, product systems and subproduct systems of Hilbert C*-modules, representing kernel Hilbert spaces, mutivariable operator theory and non-selfadjoint operator algebras. Several years ago the focus of my research was on functional equations and dynamical systems. I have written 16 research articles, which split evenly between singly authored papers and collaborative ones; I have had the pleasure to collaborate with seven excellent mathematicians. I presented my work in numerous conferences and seminar meetings, and I was lucky to be invited several times to some conferences as a plenary speaker.

B2.1 Research career potential

The reintegration period at BGU covers a period of five years: BGU's standard contract for new researchers is for an initial period of three years, followed by an option to extend the contract for a further two years. Thereafter, a decision is taken as to whether to award the researcher a permanent position. In the light of the initial careful review process of potential candidates and BGU's mentoring system, it is rare that a new researcher does not fulfil the requirements for obtaining a permanent position. Whether BGU will grant me a permanent position at the end of the reintegration period depends primarily on the quality of my research in the next five years: I am expected to establish a research group, to carry out independent research, and to supervise the work of graduate students. I therefore consider the CIG grant as an essential element in realizing my career potential, since it is regarded by the University authorities as a critical indicator of my scientific ability and my ability to attract research funding and hence of my worth as an investigator. With this support, I will be able to dedicate time to carrying out independent research and to building the publication record necessary to advance my career. Building an academic reputation is also necessary for successfully competing with senior researchers for research grants, thereby enabling me to secure additional funding for future research. Since both publications and grants are absolute requirements for a permanent position at BGU, the requested grant will therefore have a substantial impact on my integration into BGU.

B2.2 Research and technological quality of previous research

B2.2.1 Major accomplishments in noncommutative dynamics

Noncommutative dynamics is the mathematical formalism concerning time evolution in a quantum mechanical system. The central object of study is a semigroup $\phi = \{\phi_t\}_{t\geq 0}$ of completely positive maps (for short: CP-semigroup) on $B(H)$, where $B(H)$ is the algebra of all bounded operators on some Hilbert space H. When each ϕ_t is a $*$-endomorphism of $B(H)$, then we say that ϕ is an E-semigroup.

In the early '90s Bhat proved the following theorem, which is one of the cornerstones of noncommutative dynamics.

Theorem B2.1 (Bhat's Theorem [20]) *Every CP-semigroup has an E-dilation; that is, there is an E-semigroup $\alpha = \{\alpha_t\}_{t\geq 0}$ which satisfies*

$$\phi_t(T) = P_H \alpha_t(T) P_H, \tag{5}$$

for all $T \in B(H)$ and $t \in \mathbb{R}_+$, where P_H is the orthogonal projection of K onto H.

My main contribution to this field is the extension of Bhat's Theorem to CP-semigroups $\phi = \{\phi_s\}_{s\in\mathscr{S}}$ which are parameterized by some semigroup $\mathscr{S}$, rather than by the semigroup $\mathbb{R}_+$. My first significant result is that every pair of *strongly commuting* CP-semigroups has an E-dilation. First I established the unital case [66], and later the nonunital case [67].

My next significant group of results in noncommutative dynamics is the development of a general framework for studying CP-semigroups and dilations over general semigroups. Together with B. Solel, I introduced and studied the notion of a *subproduct system* [71], which is a generalization of *product systems*, introduced by Arveson in 1989. A subproduct system is a family $X = \{X(s)\}_{s\in\mathscr{S}}$ of Hilbert spaces that satisfies

$$X(s+t) \subseteq X(s) \otimes X(t).$$

To every CP-semigroup one can attach a subproduct system in a canonical way. We showed that when the CP-semigroup is *unital*, a necessary and sufficient condition for the existence of an E-dilation is that the associated subproduct system may be embedded into a *product system*. This result contains all known Bhat-type theorems for unital semigroups, and produces some new ones as well. Further, we were able to construct three commuting CP maps which have no E-dilation. This result has been expected for more than a decade. Recently, using this framework, M. Skeide and I showed that there exist three commuting *unital* CP maps for which there is no E-dilation [70]. This result was rather unexpected, and shows how the "quantum" dilation theory exhibits phenomena that have no counterpart in classical dilation theory.

B2.2.2 Major accomplishments in multivariable operator theory

My investigations on subproduct systems led me naturally to study multivariable operator theory and nonself-adjoint operator algebras (see [71] to understand why). In this area I have two major accomplishments of different natures. To describe my major accomplishments in condensed form, I will use the notations introduced in Section B1.1.1.

A d-tuple $(T_1,\dots,T_d)$ is said to be *essentially normal* if $[T_i, T_j^*]$ is compact for all i,j. A subspace $M \subseteq H^2_d$ is said to be *essentially normal* if the compression of $(S_1,\dots,S_d)$ to M is essentially normal. Arveson conjectured [12] that for any homogeneous ideal $I \subseteq \mathbb{C}[z]$, the closure of I in H^2_d is essentially normal.

My contribution to this problem is the ushering of algorithmic techniques from computational algebraic geometry into the picture [68]. I introduced the notion of *stable polynomial division*, and showed that any ideal that has the *stable division property* satisfies Arveson's conjecture. I then showed for different classes of ideals that the stable division property not only holds, but can be realized in an algorithmic manner (these are also new results in computational algebraic geometry). Thereby I gave a unified proof that Arveson's conjecture holds for most of the interesting special cases that were treated earlier in the literature. Another contribution I made here was the reduction of the entire problem to the problem of determining whether or not all ideals generated by *scalar valued quadratic forms* satisfy Arveson's conjecture.

Another problem to which I contributed, together with K. R. Davidson and C. Ramsey, is the isomorphism problem for a class of multiplier algebras.

Let V be a holomorphic sub-variety of the unit ball $\mathbb{B}_d$. Recall that $\mathscr{M}_V$ is the restriction of the multiplier algebra $\mathscr{M}_d$ of H^2_d to the variety V. A natural question is: to what extent does V determine the structure of $\mathscr{M}_V$?

In [30] we treated the case where V is a homogeneous variety. In this setting the answer is as follows: *The* **geometry** *of V completely determines the* **structure** *of $\mathscr{M}_V$.* Different interpretations of **geometry** correspond to different interpretations of **structure**. V and W are isomorphic as algebraic varieties if, and only if, the (not closed) sub-algebras of $\mathscr{M}_V$ and $\mathscr{M}_W$, respectively, generated by the compression of the shift to $\mathscr{F}_V$ and $\mathscr{F}_W$, respectively, are isomorphic as algebras. V and W are related by an isometry of $\mathbb{C}^d$ if, and only if, $\mathscr{M}_V$ and $\mathscr{M}_W$ are isometrically isomorphic (as Banach algebras). Most interestingly (at least for a wide class of varieties), the multiplier algebras $\mathscr{M}_V$ and $\mathscr{M}_W$ are isomorphic (as algebras) if, and only if, there is an invertible linear map T on $\mathbb{C}^d$ that sends V onto W and is length preserving on V. This is a subtle kind of geometry. Lastly, we also found within some low dimensional cases that the C*-algebra generated by $\mathscr{A}_V$ is completely determined by the topology of V.

The condition *there is an invertible linear map T on $\mathbb{C}^d$ that sends V onto W and is length preserving on V* is rather rigid. We proved that if V is a nonlinear hyper-surface, or if it is irreducible, then T must be a unitary (but there *are* cases when it is not). From this follows an operator-algebraic rigidity result: *Let V be irreducible. If $\mathscr{M}_V$ and $\mathscr{M}_W$ are isomorphic, then they are isometrically isomorphic, and in fact they are unitarily equivalent.*

In [31] we studied the case where the varieties in question are not necessarily homogeneous. We found that $\mathscr{M}_V$ is completely isometrically isomorphic to $\mathscr{M}_W$ if and only if W is the image of V under a biholomorphic automorphism of the ball. A similar condition characterizes when there exists a unital completely contractive homomorphism from $\mathscr{M}_V$ to $\mathscr{M}_W$. If one of the varieties is a homogeneous algebraic variety, then isometric isomorphism was shown to imply completely isometric isomorphism of the algebras.

The problem of characterizing when two such algebras are (algebraically) isomorphic was also studied. We showed that if there is an isomorphism between $\mathscr{M}_V$ and $\mathscr{M}_W$, then there is a biholomorphism (with multiplier coordinates) between the varieties. Quite surprisingly, we found a number of counter-examples showing that the converse fails in several ways.

B2.3 Independent thinking and leadership qualities

I must now describe some activities that reflect my initiative, project management, independent thinking, and leadership skills.

Innovation. I have written several papers where the introduction of an original mathematical idea was crucial. The best example is the notion of *subproduct system* introduced in [71], which is my most cited paper. Another good example is *the stable division property*, introduced in [68].

Independent thinking. I love learning from others, and reading other mathematicians' work. However, I take nothing for granted, and I think for myself. Bhat's Theorem is one of the most important results in noncommutative dynamics, so important that it has five different published proofs. When I first studied this theorem there was one particular point that I did not understand. So I read one of the other proofs. And then another, and another, and then the last one. After months of hard work I realized that all five proofs had a gap, *the same gap* in all of them. I contacted the authors, and at first there was some disagreement. I had to stand up to all the leaders of the field and insist that there is a mistake in their proofs. In the end I was able to convince them, with the help of my colleague Daniel Markiewicz. The two of us later also solved this gap [55], giving a firm foundation to this important theorem.

Leadership and collaboration. As my publication list shows, I am able to think of good problems, and to attract other researcher to work with me on them; see, for example, [53, 54] (with E. Levy), [56] (with J. E. McCarthy) or [70] (with M. Skeide). I collaborated with seven different researchers of varying levels and different fields of expertise; for example, at the time of my collaboration with them, C. Ramsey was a graduate student, D. Markiewicz was a postdoc, M. Skeide was a full professor, and K. Davidson was a University Professor, a Fellow of the Royal Society of Canada, a Fields Institute Fellow, and a world renown leader of the field. In all the projects in which I participated I was leader of some sort, either by formulating the problem and pushing towards its solution (see [53, 55]), by or designing the strategy for attack and breaking up the problem into smaller steps (see [30, 31, 71]).

Management. During my graduate studies I helped organize the Technion's seminar on functional equations and applications. In the few years since I graduated, I have taught many mathematical courses, some of them large courses with many sections (one of which I was course coordinator), some of which I ran alone. Right now I am organizing, together with my colleague Daniel Alpay, an international conference to be held at BGU in May. From all these activities I have gained and I am gaining experience in time planning, logistical organization, managing subordinates, etc.

Mentorship. During my postdoctoral fellowship at the University of Waterloo, I played the role of a non-official mentor to graduate students in operator theory. This included reading their papers and suggesting constructive criticism; serving as an expert knowledge base in certain topics and providing references, which in some cases proved of great significance for a student's work; and also simply being an address to which to the students could post questions and discuss ideas. I myself have also greatly benefited from this interaction. For example, my works [30, 31] were performed jointly with my host Ken Davidson and his PhD. student Chris Ramsey.

Initiative. In Fall 2009, upon my arrival at the University of Waterloo, I initiated an eight week special learning seminar intended for graduate students and faculty: "Product systems and subproduct systems: origins and applications". In the seminar I taught the basic notions of noncommutative dynamics and subproduct systems, and presented the cutting edge in the related dilation theory. I also arranged for a "guest star" (Michael Skeide) to come and present his own proof of the "Fundamental Theorem of Arveson Systems" (on the existence of E_0-semigroups).

Outreach. When Donal O'Shea's book *The Poincaré Conjecture: in Search of the Shape of the Universe* was translated into Hebrew, I seized the opportunity to become scientific editor, and to work closely with the translator to make sure that no mathematical content was warped during the translation process. The work was quite hard as I had to learn some of the mathematics for this purpose. The project was very fulfilling, because I took part in dissemination of deep mathematical ideas to a large audience. With hindsight, this project opened my eyes to geometry, which is an integral part of my research today.

Education. I supervised four high school students in the Technion's international summer camp *SciTech 2004*. The topic - *Cauchy and Pexider's Functional Equations in Restricted Domains* - as well as the content were planned by myself. This included delivering crash courses (of my own design) in Calculus and Functional Equations, leading problem solving sessions, as well as guiding the students through their first steps doing research. My students' final report can be accessed at http://eqworld.ipmnet.ru/en/education/edu-fe.htm.

During the reintegration period, I hope that I will continue to grow as a mathematician and as a human being, and, in particular, to sharpen my project management skills and to become a leader in my department and in the mathematical community. I will, of course, continue to mentor graduate students, but I plan also to begin to advise graduate students and postdoctoral fellows. I will participate in and lead the *Operator and System Theory Seminar* which takes place at BGU, and the *Operator Theory/Operator Algebras Seminar* which is a joint seminar of BGU, the Technion and Tel-Aviv University. I wish also to take part in the organization of conferences in my field — at the Center of Advanced Studies of BGU as well as abroad. I wish to extend my collaborations with European and American colleagues, and to initiate new research projects. Last, but not least, I hope to improve further my lecturing and teaching abilities and to contribute to the dissemination of knowledge.

B2.4 Match between the fellow's profile and project

I designed the objectives of this research proposal according to my skills and my background. Therefore I believe that my profile matches well with the project.

The proposed project requires a command of operator theory, and a strong background in function theory and in algebraic geometry. Having been a student at the Technion for nine years, I have an excellent and balanced training in all of Mathematics and, in particular, I enjoyed a very solid education in Analysis. My Ph.D. studies at the Technion, under the supervision of B. Solel, and my post graduate training in the University of Waterloo, under the supervision of K. Davidson, have prepared me well for a wide spectrum of operator theoretical challenges. Moreover, during my postdoctoral training I focused on multivariable operator theory and nonself-adjoint operator algebras, sub-fields of operator theory in which my supervisor is a world leading expert. During that period I also strengthened considerably my abilities in function theory and in algebraic geometry, and to facilitate the learning process I initiated a four week long mini-course intended for graduate students and faculty, in which I taught (and learned) that part of function theory in several complex variables that is essential to multivariable operator theory.

I have a strong professional relationship with J. McCarthy, with whom I have already collaborated [56], who is a noted authority on the interplay between function theory and operator theory. In my department in BGU I have colleagues such as D. Alpay and V. Vinnikov, which are also experts in this field, and with whom I hope to collaborate in the future.

The fact that I have worked on many different problems demonstrates that I am able to learn new subjects fast, to develop a skill and to apply it to a given problem.

Some of the results that I have published [30, 31, 68, 71] are precursors to this research proposal, and prove that I am able to solve difficult problems in the particular topic of this proposal. I also have some preliminary unpublished results [69] which show that there is promise in the direction that I propose.

I believe that I have the interpersonal skills required to recruit and train graduate students and postdoctoral fellows, which will help me carry out the research.

Most importantly, I am a very disciplined, hard working and motivated mathematician, and I am very eager to work on the problems that I described above.

B2.5 Curriculum Vitae

B2.5.1 General Information

Date of birth:	February 9, 1977
Citizenship:	Israeli
Marital Status:	Married + 6

B2.5.2 Education

Institution	Degree	Dates
Technion - I.I.T.	Ph.D.	2005-2009
Technion - I.I.T.	M.Sc.	2003-2005
Technion - I.I.T.	B.Sc.	2000-2003

B2.5.3 Fellowships, Grants, and Awards

2010 The Elisha Netanyahu Prize, Technion

2008 The Promotion of Excellence in Mathematics Prize, Technion

2007 The Pollak Fellowship, Technion

2006 The Jacobs Qualcom Fellowship/Gutwirth Prize, Technion

2005 The Haim Hanani Prize, Technion
The Szego Prize for Excellence in Teaching, Technion

2003 The Promotion of Excellence in Mathematics Prize, Technion

B2.5.4 Academic and Professional Experience

Position	Institution	Dates
Senior Lecturer	Ben Gurion University	September 2011 - current
Postdoctoral Fellow	University of Waterloo	August 1 2009 - August 31 2011

B2.5.5 Presentations at conferences

(•–invited, ○–contributed)

- ○ Workshop on Noncommutative Dynamics and Applications, July 16 - 20, 2007, Fields Institute, Canada.
- • Annual meeting of the Israeli Mathematical Union, May 29-30, 2008, Ashkelon, Israel.
- ○ Great Plains Operator Theory Symposium, June 17-22, 2008, University of Cincinnati, USA.
- • Product Systems and Independence in Quantum Dynamics, February 15-21, 2009, Oberwolfach, Germany.

- ◦ Multivariate Operator Theory Workshop, August 10-14, 2009, Fields Institute, Canada.
- Canadian Mathematical Society Winter Meeting, December 5-9, 2009, Windsor, Canada.
- Workshop on Non-commutative Dynamics and Quantum Probability, May 15-17, 2010, Regina, Canada.
- ◦ Canadian Operator Algebras and Operator Theory Symposium, June 7-11, 2010, Fredericton, Canada.
- Multivariate Operator Theory, August 15-20, 2010, BIRS, Banff, Canada.
- Product Systems and Independence in Quantum Dynamics, March 14-18, 2011, Greifswald, Germany.
- Great Plains Operator Theory Symposium, May 18-22, 2011, Arizona State Univerisity, USA.

B2.5.6 Seminars and colloquia

- During my graduate studies I contributed often (**nine times**) in the Technion's Department of Mathematics research seminars *Functional Equations Seminar*, *Operator Algebras and Operator Theory Seminar* (joint with Tel-Aviv and Ben-Gurion Universities) and *Nonlinear Analysis and Optimization Seminar*.
- Operator Algebra Seminar, January 29, 2009, University of Copenhagen.
- Analysis Seminar, September 25, 2009, University of Waterloo.
- Seminar on Free Probability and Random Matrices, January 18, 2010, Queen's University, Kingston.
- Department of Pure Math Colloquium, March 1, 2010, University of Waterloo.
- Analysis Seminar, October 8, 2010, University of Waterloo.
- Analysis Seminar, January 14, 2011, University of Waterloo.
- Quantum Information & Geometric Stats. Seminar, February 15, 2011, University of Guelph.
- Analysis Seminar, April 25, 2011, Washington University, St. Louis.
- Operator and systems theory seminar, November 28, 2011, Ben-Gurion University of the Negev, Be'er Sheva.

B2.5.7 Publications

1. K. R. Davidson, C. Ramsey and O. M. Shalit, *Operator algebras for analytic varieties*, submitted.
2. J. E. McCarthy and O. M. Shalit, *Unitary N-dilations for tuples of commuting matrices*, to appear in Proceedings of the American Mathematical Society.
3. E. Levy and O. M. Shalit, *Dilation theory in finite dimensions: the possible, the impossible and the unknown*, to appear in Rocky Mountain Journal of Mathematics.
4. O. M. Shalit, *Three remarks on a question of Acz'el*, to appear in Aequationes Mathematicae.

5. O. M. Shalit and M. Skeide, *Three commuting, unital, completely positive maps that have no minimal dilation*, Integral Equations and Operator Theory, Vol. 71 Issue 1 (2011), 55–63.
6. K. R. Davidson, C. Ramsey and O. M. Shalit, *The isomorphism problem for some universal operator algebras*, Advances in Mathematics, Vol. 228 (2011), 167–218.
7. O. M. Shalit, *Stable polynomial division and essential normality of graded Hilbert modules*, Journal of the London Mathematical Society, Vol. 83 Issue 2 (2011), 273–289.
8. O. M. Shalit, *E-dilation of strongly commuting CP-semigroups (the nonunital case)*, Houston Journal of Mathematics Vol. 35 No. 1 (2011), 203–232.
9. D. Markiewicz and O. M. Shalit, *Continuity of CP-semigroups in the point-strong topology*, Journal of Operator Theory , Vol. 64 No. 1 (2010), 149–154.
10. O. M. Shalit, *Representing a product system representation as a contractive semigroup and applications to regular isometric dilations*, Canadian Mathematical Bulletin, Vol. 53, No. 3 (2010), 550–563.
11. O. M. Shalit and B. Solel, *Subproduct systems*, Documenta Mathematica, 14 (2009), 801–868.
12. O. M. Shalit, *Conjugacy of P-configurations and nonlinear solutions to a certain conditional Cauchy equation*, Banach Journal of Mathematical Analysis, Vol. 3, No. 1 (2009), 28–35.
13. E. Levy and O. M. Shalit, *Continuous extension of a densely parameterized semigroup*, Semigroup Forum, Vol. 78, No. 2 (2009), 276–284.
14. O. M. Shalit, *What type of dynamics arise in E_0-dilations of commuting quantum Markov processes?*, Infinite Dimensional Analysis, Quantum Probability and Related Topics, Vol. 11, No. 3 (2008), 393–403.
15. O. M. Shalit, *E_0-dilation of strongly commuting CP_0-semigroups*, Journal of Functional Analysis , Vol. 255, No. 1 (2008), 46–89.
16. O. M. Shalit, *On the overdeterminedness of a class of functional equations*, Aequationes Mathematicae, Vol. 74, No. 3 (2007), 242–248.

B2.5.8 Additional professional service

1. Service on thesis committee: (Maxim Gurevich, M.Sc., 6/11/2011, Technion).
2. Referee for: Complex Anal. Oper. Theory, J. Operator Theory, J. Math. Anal. Appl., Houston J. Math., Banach J. Math. Anal., Int. J. Math. Math. Sci.
3. Reviewer for: Math. Reviews.

B3 IMPLEMENTATION

B3.1 Quality of host organization, including adequacy of infrastructures/facilities

Ben-Gurion University of the Negev (BGU) is one of Israel's largest research universities with more than 19,000 students. BGU's main campus and two smaller campuses are situated in Beer-Sheva. The University also has campuses at Sede Boqer and Eilat. The Faculty of Natural Sciences which comprises the Departments of Chemistry, Physics, Geology and Environmental Sciences, Life Sciences, Mathematics, and Computer Sciences provides a stimulating research environment that facilitates collaborations and promotes multi- and interdisciplinary research. The faculty is home to six ERC Starting Grant winners: Amir Aharoni [ERC project SULTENG; ITN project ENEFP]; Ohad Medalia [ERC project INCEL; ARCHES awardee], Michael Meijler [ERC project QUORUMPROBES]; Gonen Askenasy [ERC project Bottom-upSysChem; ITN project READ]; Taleb Mokari [ERC project NANO@ENERGY]; and Barak Weiss [Department of Mathematics; ERC project DLGAPS].

The proposed project will be performed under my leadership as principal investigator in the Department of Mathematics. The staff of the Department comprises 30 senior faculty members, 12 researchers of the immigrant scientists absorption program, 34 graduate students and 11 postdoctoral students. Members of the Department publish dozens of articles annually in prestigious mathematical journals including Inventiones Mathematicae, Duke Mathematical Journal, Advances in Mathematics, Compositio Mathematica, Mathematische Annalen, Transactions of AMS, and Israel Journal of Mathematics. The Department holds a weekly colloquium and regular seminars in various subjects in mathematics. In addition, the Department has a Center for Advanced Studies that promotes research activities in Mathematics and its applications. The Center provides support for visiting lecturers, conferences and workshops, lecture series, and postdoctoral fellows.

In my research, I intend to collaborate with colleagues in the Department, especially with the members of the strong Functional Analysis, Operator Theory and Operator Algebras group. In particular, I will cooperate with: my mentor <u>Victor Vinnikov</u> [International collaboration: UCSD, Drexel U, Virginia Tech, U Saskatchewan, Romanian Acad Inst Mathematics; Recent publications: J Funct Anal, CR Math, IEEE Trans Automat Contr, Linear Algebra Appl]; <u>Daniel Alpay</u> [International collaboration: U Iowa, Kansas State U, Inst Politecn Nacl (Mexico), U Reims, Vrije U Amsterdam, Vienna U Technol, U Groningen; Recent publications: Integr Equat Oper Th, J Funct Anal, Syst Control Lett, Stoch Proc Appl, Acta Appl Math, Math Nachr, Siam J Control Optim, J Operat Theor, Linear Algebra App, Cr Acad Sci I-Math]; <u>Daniel Markiewicz</u> [International collaboration: U Penn, U California Riverside; Recent publications: J Operat Theor, J Funct Anal, Houston J Math] and <u>Uri Onn</u> [International collaboration: Harvard U, CNRS, Penn State U, Inst Math Sci Madras; Recent publications: J Pure Appl Algebra, Cr Acad Sci I-Math, Israel J Math, Commun Algebra].

In terms of **infrastructure**, BGU will provide me with a personal office, desktop computer, and variety of online journal subscriptions. The University also has a suitable library and seminar rooms.

B3.2 Feasibility and credibility of the project, including work plan

B3.2.1 Work plan

"Ven der mentsh tracht, Got lacht" — so goes an old Yiddish proverb, meaning "Man makes plans, and God laughs". Among all the sciences, this is probably most true in mathematics. Still, we plan, and below I outline my work plan.

Scientific work plan: The first year of the project will be devoted to three lines of action. First, I will work on the first problem. This effort will split into two: to prove a theorem that one can transfer the problem to the Bergman or Hardy space, and then to perform the calculation of the essential norm in those spaces. The second line of action would be to study extremely carefully the current literature in multivariable operator theory and operator algebras. The third task I am going to undertake in the first year is to give an advanced courses in multivariable operator theory, operator algebras, functions of several complex variables and reproducing kernel Hilbert spaces — this will allow me to recruit graduate students to work with me.

In the second year I hope to recruit a postdoctoral fellow and a graduate student to start working with me. Having this critical mass at hand, we will run a weekly learning seminar in the topics that we will think to be most promising for the project. I will also begin working on Problems B1.2 and B1.3, splitting some of the work between myself and my research assistants.

By the end of the second year I hope to have solved Problem B1.2. This success would imply the solution of the easier version of Problem B1.4. In the third year we will begin to concentrate more fully on Problem B1.3. Depending on our progress, one of my research assistants will start to write a computer program that will help us test whether the stable division property holds in certain cases. This will help us in gaining intuition by considering examples, in collecting experimental evidence for the validity of stable division, or in spotting candidates for a counter example.

The completion of the project will take place in the fourth year of the project. The focus will be on finishing up Problem B1.3, as the difficult version of Problem B1.4 is a corollary.

Other aspects of the work plan: I hope that within the first two years of the contract I will be able to attract new collaborators to work with me on this project. I will make a special effort to recruit a postdoctoral fellow and graduate students to work with me.

Attracting strong graduate students to my area requires that I teach advanced courses on the topics of multivariable operator theory, operator algebras, functions of several complex variables and reproducing kernel Hilbert spaces. After recruiting graduate students I will have to continue their training, which will begin in the second year and continue until the end of the contract and perhaps beyond that. This training will consist of weekly meetings, assigned reading, learning seminars and student seminars. My goal is that graduate students who work with me, aside form helping me complete my research project, will gain expertise that will enable them to become *independent* researchers in this field. Therefore, some room will be given for the graduate students to pursue their own directions and to propose their own ideas within the research project that I shall manage.

During the second and the third years, I will start publishing the preliminary results and presenting them at international conferences and seminars. I expect to make the main progress in the proposed research during these two years.

The last year will be devoted to completing the project, writing the final results, and to considering the new directions in which the success of this project will lead us.

B3.2.2 Career development plan

My goal is to obtain a permanent position at BGU that will enable me to build and lead a dynamic group conducting research in mutiariable operator theory and operator algebras. In the decision process as to whether to grant a permanent position, BGU examines a researcher's portfolio of activities for a number of key successes, among which are: publications record (both as a co-author and as a single author), funding (especially the CIG grant and other EU funding), ability to manage research, number and quality of graduate students, teaching abilities, as well as national and international collaborations. In my career development plan, the CIG grant will enable me to conduct a successful project that will lead to the publication of papers in prestigious journals and to the acquisition of additional grants. It will enable me to present my work at international conferences, where I will be able to make contact with leading researchers in the field with a view to future cooperative EU-funded research and to the transfer of knowledge within the ERA. The new advanced courses on mutiariable operator theory and operator algebras that I intend to teach in the Department will serve to attract graduate students. BGU, for its part, will place at my disposal the necessary management support and training in complementary skills (see Section B3.3). In addition, the University has put in place a career guidance mentoring plan in which a senior faculty member serves as an advisor assisting the new faculty member to identify and exploit external and internal research resources; to understand the campus and the local community; and to prepare documents, reports, and proposals.

B3.2.3 Intellectual property rights

In keeping with the European Charter for Researchers, institutional and personal IP rights are protected in line with BGU's Ethical Code. Researchers receive 40% of the royalties on their inventions. New staff members receive one–on–one instruction on patents from B. G. Negev Technologies (BGN), the University's commercialization arm. BGN also aids new researchers in the exploitation of their results through the Company's established procedures and relationships with commercial enterprises the world over.

B3.2.4 Budgetary breakdown covering the duration of the project

The table below describes the *Total Estimated Project Costs (TEPC)* and *EC Requested Contribution (ERC)*.

	1st year(€)		**2nd year(€)**		**3d year(€)**		**4th year(€)**	
	TEPC	**ERC**	**TEPC**	**ERC**	**TEPC**	**ERC**	**TEPC**	**ERC**
Salary of PI	15,000	8,000	15,000	8,000	15,000	8,000	15,000	8,000
Personnel costs	8,000	4,000	33,000	9,727	33,000	9,727	15,000	9,727
Travel costs	8,000	5,727	6,000	4,000	6,000	3,700	6,000	4,000
Equipment	3,000	3,000			1,000	500		
Miscellaneous	1,000	2,000	1,000	1,000	1,000	800	1,000	1,000
Overhead (10%)	3,500	2,273	5,500	2,273	5,600	2,273	3,700	2,273
Total cost	**38,500**	**25,000**	**60,500**	**25,000**	**61,600**	**25,000**	**40,700**	**25,000**

B3.2.5 Budget justification

Personnel: The contract signed with the University is for full-time employment. The salary of the PI is mostly covered by internal sources. In this project 30% of the PIs salary will be reported as cost.
Personnel costs: Funds are requested to cover the salaries for employment of two research assistants: a Ph.D. student (three and a half years, part time) and of postdoctoral fellow (2 years). The Ph.D. student will work with me towards the solution of Problems B1.1 and B1.2. According to the contingency plan, the PhD student may also tackle the *algorithmic* approach to Problem B1.3. The postdoctoral fellow will join the project in the second year, and will concentrate on the *analytic* approach to Problem B1.3.
Travel costs: : Funds are requested to partially cover the expenses of the PI and his research assistants to travel to conferences in Europe and North America in order to present the results and to establish future research collaboration opportunities.
Equipment: Funds are requested to cover the costs of computers and software for the PI and his prospective research assistants.
Miscellaneous: Funds are requested to cover the costs of purchasing books, printing, office supplies, communication expenses. Funds are also requested to cover some expenses involved with organizing a seminars, such as reimbursing travel costs of invited speakers.

B3.3 Management: Practical arrangements for the implementation and management of the project

B3.3.1 Contingency plan:

One cannot think of a worthwhile goal that is not challenging and in some sense risky. The project that I am proposing is no exception, and I should not be so so arrogant as to be absolutely certain that I will succeed. The project is made up of four different problems, some of which are independent of the others, as part of my contingency plan. Let me describe what else I plan to do in case I encounter obstacles on my way.

A general remark is that in mathematics an *obstruction* to the solution of a problem can be a *wonderful* thing. If some problem is unsolvable, perhaps there is some deep reason why this is so, and it may pay off to search for this deep reason, and once it is found, to investigate it. This often leads to a better understanding of the original problem: once the nature of the obstruction is understood, one can understand under what circumstances is the problem solvable an how to solve it in those cases.

Two things can go wrong with my plan for solving Problem B1.1. It might be that the computation of the essential norm in Drury-Arveson space cannot be transferred to the computation over the Hardy/Bergman spaces. In this case I will have to think of methods to compute the essential norm in the original space. Also, I will study what is it that prevents us from transferring the computation of the essential norm to the other spaces.

Another thing that can go wrong with my plan for Problem B1.1, is that the computation of the essential norm in the Hardy/Bergman spaces will be intractable. This is unlikely, but in this case I will have to concentrate on spaces cases in which it is tractable.

Problem B1.2 might unluckily turn out to be just as hard as Problem B1.4. This is one of the reasons why I thought of two different approaches to tackling problem B1.4. Thus, this project has a chance of being a great success even in the case where half of the goals (say Problems B1.1 and B1.2, or Problem B1.3, separately) are not fulfilled.

In case that I will not be able to make any progress in my approach to Problem B1.3, I plan to reconsider the algorithmic techniques that I studied in the past. Moreover, perhaps different norms on $\mathbb{C}[z]$ might turn out to be appropriate for the study of stable division, and I might try to find a more suitable norm where an essential Hilbert's Basis Theorem can be proved.

B3.3.2 Practical arrangements for management of the research project:

The legal, contractual, and financial aspects of the CIG grant will be the responsibility of BGU's Research and Development Authority (RDA). This dedicated unit handles all aspects of grant administration (negotiation, budgeting, financial reporting, and overall supervision) of all research contracts performed under University auspices. The RDA will ensure full and complete contractual compliance, guaranteeing that the project is carried out in strict accordance with EU guidelines and subject to University regulations. In practical terms, the RDA will ensure that all reporting requirements will be entered into the computerized financial management system of the University and that progress of the project will be closely and systematically monitored. The RDA will assign to the project both a Grants Coordinator and a Budget Officer, who will work under the supervision of senior personnel. Financial reports will be prepared by the Budget Officer and then reviewed/approved by the RDA's Financial Comptroller.

B3.3.3 Management strategy according to the "European Charter for Researchers":

BGU's management strategy is based on the principles of this Charter as follows.

• *Recognition of researchers as professionals on a career path,* • *Promotion of career development,* • *Recognition and promotion of the value of mobility,* • *Access to career advice and to research training and development,* • *Supervision for early stage researchers*: BGU has acquired considerable experience in integrating young scientists returning to the country from prestigious institutions abroad. To aid in their career development, the University provides a mentoring framework tailored to each young researcher and offers a number of programs for training in complementary skills, which include a course in academic teaching given by BGUs Center for Excellence in Teaching and regular training and career development seminars on topics such as grant writing, presentation skills and multimedia-assisted teaching.

• *Stability and permanence of employment,* • *Evaluation systems,* • *Funding and salaries,* • *Legal working conditions*: See Section B4.2.

• *Protection of intellectual property rights*: See Section B3.2.3.

• *Teaching duties*: In their first year at BGU, researchers are given reduced teaching loads, which comply with national regulations and do not exceed 8 hours per week, thereby leaving the young researcher with ample time for academic pursuits.

• *Co-authorship*: Co-authorship, as a reflection of the ability of the young researcher to perform collaborative research, is viewed positively by BGU authorities. At the same time, BGU recognizes that young researchers should also publish independently of their mentors.

• *Gender balance*: BGU attempts to reach a gender balance at all staff levels. This effort is promoted by the actions of the University's President, the only woman president of an Israeli university.

• *Stimulating and safe research environment*: BGU has three dedicated administrative departmentsSafety and Health in the Workplace, Radiation Safety, and Securityand abides by all national health and safety regulations pertaining to research and to working conditions.

• *Stimulating research environment*: see Section B3.1.

• *Flexible working conditions*: Researchers enjoy academic freedom and flexible working conditions that enable them to combine career and family.

• *Non-discrimination*: BGU's equal opportunity policy guarantees it is open to all people, regardless of race, religion, gender, sexual orientation, national origin, marital status, disability, age, or political views.

• *Complaints/appeals procedures*: Employees may express dissatisfaction and appeal decisions of any kind. BGU has in place several avenues of appeal, depending on the issue in question. In addition, any faculty member or student can request that the University's comptroller investigate issues related to research, career development or any other aspect of university life.

• *Participation in decision-making bodies*: The participation of young researchers on consultation and decision-making bodies enables them protect and promote their interests as professionals and to actively contribute to the workings of the University. At the same time, BGU seeks to protect young researchers by not requiring that they sit on committees whose demands would significantly reduce their time available for research.

B3.3.4 Recruitment strategy according to the "Code of Conduct for the Recruitment of Researchers":

BGU recruits early stage researchers in compliance with the above Code: Recruitment procedures are conducted by balanced panels that are open, efficient and transparent. Candidates are judged on the basis of qualitative and quantitative criteria and qualifications and are not penalized for career breaks. On the contrary, multidimensional career tracks and mobility experience are viewed favourably.

B4 IMPACT

B4.1 Contribution to research excellence by attracting and retaining a first class researcher

By hiring me BGU, Israel, and the EU will benefit as follows:

- During my postdoctoral studies at the University of Waterloo, I significantly sharpened my skills in operator theory and in the theory of operator algebras. I took several advanced courses, I participated in research seminars and in learning seminars, and I delivered two mini-courses. I established good contacts with the members of the Pure Math Department at the University of Waterloo. By being situated in Canada for two years, I had many opportunities to participate in conferences and workshops in North America, and I took advantage of these, making more contacts. I also gained experience at conducting collaborative and independent research at a high level. Returning to the European Research Area (ERA), I bring the knowledge I gained and the connections I established in North America.

- I will transfer my knowledge and experience to the Israeli and European Mathematical Communities by collaborating with Israeli and European colleagues, giving talks at seminars, summer schools, and conferences, and teaching graduate courses at BGU and mini courses at summer schools. In this way, by returning from North America to ERA, I will contribute to Israeli and European scientific excellence.

- In my years abroad I have established good connections with members of the American, Canadian, Asian and European mathematical communities, and I have collaborated with some of them in the past, and plan to collaborate with some in the near future. I am confident that the European mathematical community will benefit from future collaborations and knowledge transfer from my North–American coauthors and from myself.

- The problems that I am working on are likely to attract prospective graduate students and postdoctoral researchers who may otherwise prefer to study in the United States, Canada or Asia. I intend to teach a variety of graduate and undergraduate courses and to organize a special seminar for graduate students at BGU. The up–and–coming generation of Israeli operator-algebraists and operator-theorists will be trained, in part, by me, and this new generation will also contribute to European excellence and competitiveness.

- I will continue to work on important problems, to study examples and to develop theory in operator theory and in operator algebras. My excellent track record proves that I am capable of contributing this way to European excellence and competitiveness.

B4.2 Potential and quality of the researcher's long term professional integration in Europe

The prospects of the researcher for lasting professional integration in Europe – at BGU – are governed by the "European Charter for Researchers", as follows:

• *Legal working conditions, including type and length of work contract and full-time dedication;*
• *Funding and salaries, including attractiveness of the remuneration package*: All BGU researchers are employed according to contracts that comply with national and sectorial conditions in terms of employment stability and salary scales. The contract signed with the University is for full–time employment. Researchers also have the ability to supplement their salaries on the basis of achievements in obtaining competitive research grants and industrial funding. Social security provisions (sickness

and parental benefits, pension rights and unemployment benefits) are in place and are determined by national legislation.

• *Stability and permanence of employment, including prospects for long-lasting employment;*

• *Evaluation systems*: Young researchers are appraised at three and five years by committees composed of local and international experts in their fields according to the following criteria: research progress, publications record, ability to obtain competitive funding, ability to establish and supervise a research group where relevant, national and international collaboration, teaching ability, and public awareness and education outreach activities. It is the experience of the University that such a system in combination with the mentoring framework ensures that the early stage researcher completes the five-year evaluation period successfully. Thereafter, the researcher is offered a permanent position.

This project will be important for my career development, because it will build the capacity to undertake large-scale research projects and will enable me to establish a new field of research at BGU, which is one of the reasons that I was recruited. My contract is for full time employment, and as said above is for 3 years with a further extension of 2 years, at the end of which I will be offered a permanent position. The remuneration package is in line with national contracts, and will be even increased according to research funding from external sources such as this program.

B4.3 Potential of transferring knowledge to the host organization

There will be three channels by which I will transfer my knowledge to the Department of Mathematics at BGU.

1. I will collaborate with members of the department. I have already started to work with Dr. Daniel Markiewicz (on operator algebras) and I plan to collaborate with Prof. Daniel Alpay and Prof. Victor Vinnikov (on multivariable operator theory and connections to function theory and algebraic geometry). There are also some other members of the Department of Mathematics and of the Department of Electrical Engineering with whom I might collaborate in the future.

2. I will give seminar talks and colloquia on my research and on advanced topics that I wish to learn and teach.

3. I will teach and advise students at all levels. I will teach undergraduate courses, advanced graduate courses, and I will lead student learning seminars on specific cutting edge topics. I will recruit graduate students and and I will give them personal guidance.

B4.4 Capacity to develop lasting co-operation and collaborations with the other countries

In my short career I have been to many conferences and I have ties with researchers from all over the world, including Europe, Asia and North America. In particular, during my stay in North America I met many American and Canadian mathematicians. Some of these are senior mathematicians, leaders of the field, and some were graduate students or postdocs at the time that I met them, and are the future of the field. I plan to nurture the connections that I have made, and to make new connections as well.

Let me mention my most important ties.

I have collaborated with my postdoctoral supervisor Prof. Kenneth Davidson (University of Waterloo, Waterloo, Canada) and with his Ph.D. student Christopher Ramsey, and I published two papers with them on the isomorphism of operator algebras. My ties with Prof. Davidson are invaluable to me, and I plan to keep them strong.

During my stay at the University of Waterloo I also established close connections with some graduate students, Adam Fuller, Ryan Hamilton and Matthew Kennedy. We still correspond and discuss mathematics. Dr. Kennedy is now an Associate Professor at Carleton University and is a rising star. Adam Fuller and Ryan Hamilton are about to graduate and be launched into their careers. Having connections with these up–and–coming mathematicians will serve me well.

When I participated in an international workshop in 2010 in BIRS (Banff, Canada) I met Prof. John McCarthy (Washington University, St. Louis, USA), and we continued to be in touch after that. Prof. McCarthy invited me to visit his university, where I gave a seminar talk, and this visit resulted in a joint paper in multivariable operator theory. I plan to continue to work together with Prof. McCarthy, and he has also expressed his interest in continuing our partnership.

When I was at a conference at the Fields Institute (Canada) in 2007 I met Michael Skeide (University of Molise, Italy), and we have been in touch since then. He was invited to visit me at the University of Waterloo, and when he was on sabbatical at Queen's University, Kingston, he invited me to visit him. We have been working together on many problems. We have one joint paper and one very active project running.

I met B. V. R. Bhat (Indian Statistical Institute, Bangalore, India) in 2009 in a workshop in Oberwolfach (Germany). Recently, when Prof. Bhat was visiting Israel, I discussed (together with my colleague Daniel Markiewicz from BGU) some problems in noncommutative dynamics with him. We hope that these discussions will ripen to a collaboration.

B4.5 Plans for dissemination and exploitation of results

In order to disseminate my results I will take three actions:

1. <u>Talk</u>. I will talk at seminars and conferences. I will talk while my work is in progress, in order to get feedback and in order to attract people to work on my problems; and I will give talks after my work (or parts of it) are complete, so that my ideas can make their impact on others. BGU has a special fund to allow me to visit international conferences in order to lecture and meet other mathematicians, and I plan to make full use of it. I will also use the funding offered by CIG for this purpose.

2. <u>Write</u>. I will write up my results as papers, and I will submit them to prestigious, peer-reviewed journals, so that they are refereed, indexed and reviewed. Thus my results will become an organ in the living body of mathematical knowledge.

3. <u>Post on-line</u>. I will post my papers on the preprint server ArXiV, and I will put a link on my homepage, so that every person may find and read the papers which I write. All of my results will be accessible to anyone in the world who may be interested in them.

B4.6 Impact of the proposed outreach activities

I will undertake outreach activities to explain my work to the general public — as recommended in *The European Charter for Researchers and the Code of Conduct for the Recruitment of Researchers* — on both a personal level and through the framework of the outreach activities of BGU to school goers. The Department of Mathematics at BGU has a program for gifted junior-high-school students called "Maths for Gifted Pupils". I plan to give a lecture in this program and to describe my work. Besides giving the students a little knowledge about what is happening in the frontier of mathematics, I hope that I will be able to convey the excitement involved in doing mathematical research.

On a personal level I intend to contribute in several ways. First, I will write Wikipedia entries on Arveson's conjecture and on my main results. I will also edit related articles in my fields of speciality, operator theory and operator algebras. I will open a blog devoted to operator theory and operator algebras. There are many mathematical blogs out there on the web, written by leading mathematicians in several areas, but it seems that there is no blog that puts an emphasis on operator theory and operator algebras. My blog will be written in English, and it is my hope that students from all over the world who are considering advanced studies in mathematics might get some encouragement and inspiration from reading my blog. Finally, I plan to write an article about mathematics and mathematics education for the Israeli homeschooling newspaper “Beofen Tiv’i”. In all my outreach activities I will acknowledge the contribution of the Marie Curie program.

B5 ETHICAL ISSUES TABLE

There are no ethical issues involved in the proposed project. It includes no work with cells (human or other), animals, private data/information etc. No part of the project will be worked out in a developing country. Finally the proposed project has no potential dual use, and its results can not be used in military industry, or for any other military purposes.

	Research on Human Embryo/Foetus	**YES**	**Page**
*	Does the proposed research involve human Embryos?		
*	Does the proposed research involve human Foetal Tissues/Cells?		
*	Does the proposed research involve human Embryonic Stem Cells (hESCs)?		
*	Does the proposed research on human Embryonic Stem Cells involve cells in culture?		
*	Does the proposed research on Human Embryonic Stem Cells involve the derivation of cells from Embryos?		
	I CONFIRM THAT NONE OF THE ABOVE ISSUES APPLY TO MY PROPOSAL	X	

	Research on Humans	**YES**	**Page**
*	Does the proposed research involve children?		
*	Does the proposed research involve patients?		
*	Does the proposed research involve persons not able to give consent?		
*	Does the proposed research involve adult healthy volunteers?		
	Does the proposed research involve Human genetic material?		
	Does the proposed research involve Human biological samples?		
	Does the proposed research involve Human data collection?		
	I CONFIRM THAT NONE OF THE ABOVE ISSUES APPLY TO MY PROPOSAL	X	

	Privacy	**YES**	**Page**
	Does the proposed research involve processing of genetic information or personal data (e.g. health, sexual lifestyle, ethnicity, political opinion, religious or philosophical conviction)?		
	Does the proposed research involve tracking the location or observation of people?		
	I CONFIRM THAT NONE OF THE ABOVE ISSUES APPLY TO MY PROPOSAL	X	

	Research on Animals	**YES**	**Page**
	Does the proposed research involve research on animals?		
	Are those animals transgenic small laboratory animals?		
	Are those animals transgenic farm animals?		
*	Are those animals non-human primates?		
	Are those animals cloned farm animals?		
	I CONFIRM THAT NONE OF THE ABOVE ISSUES APPLY TO MY PROPOSAL	X	

	Research Involving Developing Countries	YES	Page
	Does the proposed research involve the use of local resources (genetic, animal, plant, etc)?		
	Is the proposed research of benefit to local communities (e.g. capacity building, access to healthcare, education, etc)?		
	I CONFIRM THAT NONE OF THE ABOVE ISSUES APPLY TO MY PROPOSAL	X	

	Dual Use	YES	Page
	Research having direct military use		
	Research having the potential for terrorist abuse		
	I CONFIRM THAT NONE OF THE ABOVE ISSUES APPLY TO MY PROPOSAL	X	

	Consistency with part A	YES	Page
	I CONFIRM THAT THE INFORMATION GIVEN IN THIS TABLE IS CONSISTENT WITH THE INFORMATION PROVIDED ON ETHICS IN PART A, PAGE A1	X	

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PART B

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