RESEARCH ACHIEVEMENTS AND PEER RECOGNITION

- **Research achievements**

I am a unique mathematician with an eclectic palette of research interests and capabilities, an outstanding ability to collaborate and lead and a record of discovering penetrating mathematics based on innovative ideas. My main contributions, described from a high-level view, indicate the path that has led me to my current outlook.

Subproduct systems and dilation theory for multiparameter CP-semigroups

My PhD focused on dilating multiparameter CP-semigroups on von Neumann algebras to $*$-endomorphism semigroups. I proved a positive two-parameter result [41, 42] and settled a continuity question [34], correcting a mistake in papers by Arveson, Bhat, Muhly, Skeide, and Solel. In my top PhD paper [46] with Solel, we introduced *subproduct systems* and their representations, giving necessary and sufficient conditions for dilations and solving Bhat's open problem by producing three commuting CP maps with no dilation. With Skeide [45], we went far beyond [46] by developing new tools for CP-semigroups over Ore semigroups, proving that a unital CP-semigroup dilates to an E-semigroup if and only if its subproduct system embeds into a product system. We used these methods to settle existence, uniqueness, and topological issues in both unital and non-unital cases, and even in the well-studied one-parameter case we improved Arveson's equivalence-of-minimality results, proving them also in the non-unital case he had conjectured two decades earlier.

Outstanding papers:
1. O.M. Shalit and B. Solel, *Subproduct systems*, **Documenta Mathematica**, Vol. 14 (2009).
2. O.M. Shalit and M. Skeide, *CP-semigroups and dilations, subproduct systems and superproduct systems: the multi-parameter case and beyond*, **Dissertationes Mathematicae**, Vol. 585 (2023).

Classification of Hardy algebras of subproduct systems and complete Pick algebras

Subproduct systems were introduced as a tool for analyzing CP-semigroups and dilations, but they also naturally give rise to C*- and nonselfadjoint operator algebras. With Davidson and Ramsey [12], we showed that two Hardy algebras of subproduct systems are isometrically isomorphic exactly when the subproduct systems are. Restricting to commutative subproduct systems yields algebras of multipliers $\mathscr{M}_V$ on homogeneous varieties $V \subset \mathbb{C}^d$. In [12] and subsequent work by Hartz [22], we classified them up to biholomorphism, drawing on operator theory, several complex variables, and algebraic geometry. The main result is that if $V, W \subset \mathbb{C}^d$ are homogeneous varieties, then $\mathscr{M}_V$ and $\mathscr{M}_W$ are isometrically isomorphic if and only if V and W are biholomorphic, and this happens if and only if there is an invertible linear map $A\colon \mathbb{C}^d \to \mathbb{C}^d$ that maps V onto W.

This led me to study the isomorphism problem for complete Pick algebras. In [13] with Davidson and Ramsey we showed that $\mathscr{M}_V \cong \mathscr{M}_W$ completely isometrically if and only if $V \cong W$ via an automorphism of $\mathrm{Aut}\,\mathbb{B}_d$. The homogeneous case suggested that $\mathscr{M}_V \cong \mathscr{M}_W$ algebraically is equivalent to $V \cong W$ biholomorphically, but with Hartz and Davidson we showed this to be false even for analytic discs [10]. In [32] we showed that for nice one dimensional subvarieties of the ball $\mathscr{M}_V \cong \mathscr{M}_W$ algebraically if and only V and W are biholomorphic. The main part of the proof was to show that $\mathscr{M}_V = H^\infty(V)$ with equivalent norms. As a surprise reward, this yielded a strong Henkin-type extension theorem with multiplier norm control.

In [13] we showed that if V and W are a finite union of irreducible or discrete varieties then $\mathscr{M}_V \cong \mathscr{M}_W$ as algebras implies that $V \cong W$ via a *multiplier biholomorphism*. However, in general multiplier biholomorphism does not imply that the algebras are isomorphic [10]. The reason: *multiplier biholomorphism is not an equivalence relation!* Since algebraic isomorphism is an equivalence relation for multiplier algebras, this raised the question whether the variety V is a complete invariant for the algebra $\mathscr{M}_V$; if it is, then what are the morphisms of the corresponding category; and if it isn't, then what is the correct invariant?

Outstanding papers:
1. K.R. Davidson, C. Ramsey and O.M. Shalit, *The isomorphism problem for some universal operator algebras*, **Advances in Mathematics**, Vol. 228 (2011).
2. K.R. Davidson, C. Ramsey and O.M. Shalit, *Operator algebras for analytic varieties*, **Transactions of the American Mathematical Society**, Vol. 367 (2015).

Algebras of bounded NC functions on subvarieties of operator balls

Around 2015 I realized that all the algebras I studied can be viewed as algebras of bounded noncommutative (NC) functions on subvarieties of the NC row unit ball. I envisioned that the correct invariant should be the corresponding NC variety, and that the morphisms should be an appropriate family of NC holomorphic mappings. This program was highly successful. In [37, 38] with Salomon and Shamovich, we initiated the classification program for algebras of bounded NC functions on subvarieties of the NC row ball $\mathfrak{B}_d$. The main results, modulo some details, are that $H^\infty(\mathfrak{V}) \cong H^\infty(\mathfrak{W})$ completely isometrically if and only if an automorphism of $\mathfrak{B}_d$ maps $\mathfrak{V}$ to $\mathfrak{W}$, and completely bounded isomorphisms correspond to bi-Lipschitz NC biholomorphisms of the similarity envelopes. Stronger results hold in homogeneous or commutative cases, covering subproduct systems and multiplier algebras. These results required developing delicate NC function-theoretic tools, such as NC analogues of the maximum modulus principle, Cartan's uniqueness theorem, various NC versions of the Nullstellensatz, and the Vesentini–Schwarz lemma, as well as spectral radius theory. The work triggered significant subsequent research in NC function theory [2, 5, 27, 28]. Recently, with Sampat [39, 40], we initiated the program to extend the theory to operator balls, uncovering new and unexpected phenomena which emerge from the underlying operator space structure which lie at the heart of my current research program.

Outstanding papers:
1. G. Salomon, O.M. Shalit and E. Shamovich, *Algebras of bounded noncommutative analytic functions on subvarieties of the noncommutative unit ball*, **Transactions of the American Mathematical Society**, Vol. 370 (2018).
2. G. Salomon, O.M. Shalit and E. Shamovich, *Algebras of noncommutative functions on subvarieties of the noncommutative ball: the bounded and completely bounded isomorphism problem*, **Journal of Functional Analysis**, Vol. 278 (2020).
3. J. Sampat and O.M. Shalit, *On the classification of function algberas on subvarieties of noncommutative operator balls*, **Journal of Functional Analysis**, Vol. 288 (2025).

NC convexity, matrix convex sets and the new dilation theory

NC function theory has led me back to dilation theory via the framework of NC convexity and matrix convex sets. In [9] with Davidson, Dor-On and Solel, we proved foundational results on matrix range inclusion and CP interpolation, and established that the matrix range is a complete invariant of the operator system. With Passer and Solel [36], we computed sharp dilation and inclusion constants, improving bounds of Helton–Klep–McCullough–Schweighofer [26] and Fritz–Netzer–Thom [15]. These papers were well received and highly cited by papers in pure mathematics and quantum information theory [4, 6, 7, 11]. With Gerhold, Pandey and Solel [16, 20], besides computing sharp dilation constants, we revolutionized dilation theory. Whereas dilations are traditionally used to view a general object as part of a simpler and better understood object, we started using dilations symmetrically as a measure of distance between tuples of operators. This was applied to von Neumann's problem on boundedly perturbing the position and momentum operators from quantum mechanics to a pair of commuting selfadjoints [21], improving the constant obtained by Haagerup–Rørdam. In our recent breakthrough paper [18], Gerhold and I used our dilation techniques to show the stability of ergodic commutation relations, which can be viewed as a counterpart of Lin's theorem for q-commuting tuples. In [19] Gerhold and I used methods from random matrix theory and free probability to determine what a matrix range almost surely looks like, and very recently, with Scherer, we combined these tools with numerical methods in an experimental work for determining the universal dilation constant [17].

Outstanding papers:
1. K.R. Davidson, A. Dor-On, O.M. Shalit and B. Solel, *Dilations, inclusions of matrix convex sets, and completely positive maps*, **International Mathematics Research Notices**, Vol. 2017 (2017).
2. B. Passer, O.M. Shalit and B. Solel, *Minimal and maximal matrix convex sets*, **Journal of Functional Analysis**, Vol. 274 (2018).
3. M. Gerhold, S. Pandey, O.M. Shalit and B. Solel, *Dilations of unitary tuples*, **Journal of the London Mathematical Society**, Vol. 104 (2021).
4. M. Gerhold and O.M. Shalit, *Dilation distance and stability of ergodic commutation relations*, under revision for **Journal of Functional Analysis**.

Multivariable operator theory

I pioneered a very useful finitary approach to dilation theory (see [33, 35] with Levy and McCarthy) which has been developed to perfection by Hartz and Lupini [23], and gets cited often by papers on quantum computation [8, 14, 25]. In [43], I introduced methods from computational algebraic geometry to bear on Arveson's essential normality conjecture. This work inspired Kennedy's contribution [29], which then led our joint work [30], in which we coined *the geometric Arveson-Douglas conjecture* and obtained the best existing result for highly reducible varieties: the essential normality for quotients corresponding to unions of subspaces. In another influential paper [31], Kennedy and I proved that key consequences of the conjecture hold for all graded quotients, and that for a every quotient module the conjecture holds if and only if it is hyperrigid in the sense of Arveson [3]. I have also contributed to multivariable operator theory by proving with Hartz and Richter a dimension-dependent von Neumann inequality for row contractive matrix tuples [24], a result that answered several questions in the literature, including an open problem of Aleman, Hartz, McCarthy and Richter [1] about subhomogeneity of commutative function algebras, as well as an open problem from [37] about uniform continuity of NC functions. My recent breakthrough paper [44] with Shamovich features a spectral radius function $\rho_{\mathscr{E}}$ associated with every operator space $\mathscr{E}$, based on the Haagerup tensor product. We prove that when applied to a tuple $A = (A_1, \dots, A_d)$ of operators, $\rho_{\mathscr{E}}(A) < 1$ precisely when $A_1, \dots, A_d$ are jointly similar to a point in the open unit ball over $\mathscr{E}$. This was applied to determine the domain of definition of NC rational functions on the operator space dual $\mathscr{E}^*$. We expect our spectral radius to play a pivotal role in the success of the classification program for algebras of bounded NC functions.

Outstanding papers:
1. M. Kennedy and O.M. Shalit, *Essential normality, essential norms and hyperrigidity*, **Journal of Functional Analysis**, Vol. 268 (2015).
2. M. Hartz, S. Richter and O.M. Shalit, *von Neumann's inequality for row contractive matrix tuples*, **Mathematische Zeitschrift**, Vol. 301 (2022).
3. O.M. Shalit and E. Shamovich, *A spectral radius for matrices over an operator space*, **Advances in Mathematics**, Vol. 479 (2025), 110449.

- **Peer recognition**

My work is well known by my peers, highly cited, and has influenced the work of several outstanding mathematicians. The best way to verify this is to consider my Google Scholar page, which currently lists 1291 citations, h-index 22, and i10 index 30.

Some of my papers have been published in top journals, such as *Advances in Mathematics*, *Journal of the London Mathematical Society* and *Transactions of the American Mathematical Society*. I publish regularly in *Journal of Functional Analysis* — the top journal in my field. My chief consideration when choosing a journal is the existence of an editor who is knowledgeable in the field and would know what to do with my paper.

I have been awarded five competitive research grants: three consecutive Israel Science Foundation (ISF) grants, one German Israeli Foundation (GIF) grant, and one Marie-Curie Career Integration (CIG) grant. I also won a couple of Israel Science Foundation workshop grants and one internal Technion-UWaterloo grant.

I have been invited to give talks in numerous conferences and workshop, including (semi-)plenary talks at GPOTS (once), COSY and IWOTA (multiple times), which are among the most important conferences in operator theory and operator algebras. I am especially proud to have been invited to give lecture series and mini-courses on six occasions, at the Universities of Athens, Copenhagen, and Manitoba (as part of COSY), at NISER Bhubaneswar, India, at Washington University in St. Louis and at Oberwolfwach.

I serve on the editorial board of the journals Acta Sci. Math. (Springer) and Infin. Dimens. Anal. Quantum Probab. Relat. Top. (World Scientific), and in the years 2016 – 2020 I was an editor for the Banach J. Math. Anal. (Duke University Press/Springer).

Among other excellence prizes and fellowship that I was awarded during my studies, the most notable I received is the Elisha Netanyahu Prize, which is awarded annually to a single outstanding doctoral student.

My extensive network of collaborators, including many repeat collaborations, is another indication of my standing in the field; some of the them are among the very best in their areas. I have been privileged to serve as postdoc advisor to eleven young mathematicians from around the world, several of whom are truly exceptional, often at the recommendation of their mentors. I am also grateful for the opportunity of supervising ten graduate students and eleven undergraduate research students.

References

[1] Aleman, A., Hartz, M., McCarthy, J. E. and Richter, S. (2022) Multiplier tests and subhomogeneity of multiplier algebras. *Doc. Math.*, 27, 719–764

[2] Arora, P., Augat, M., Jury, M. and Sargent, M. (2024) An optimal approximation problem for free polynomials. *Proc. Amer. Math. Soc.*, 152(2), 455–470

[3] Arveson, W. B. (2011) The noncommutative Choquet boundary II: hyperrigidity. *Israel J. Math.*, 184, 349–385

[4] Aubrun, G., Lami, L., Palazuelos, C. and Plávala, M. (2021) Entangleability of cones. *Geom. Funct. Anal.*, 31(2), 181–205

[5] Belinschi, S. T. and Shamovich, E. (to appear) Iteration theory of noncommutative maps. *J. Noncomm. geom.*

[6] Bluhm, A. and Nechita, I. (2018) Joint measurability of quantum effects and the matrix diamond. *J. Math. Phys.*, 59(11), 112202, 27

[7] Bluhm, A. and Nechita, I. (2020) Compatibility of quantum measurements and inclusion constants for the matrix jewel. *SIAM J. Appl. Algebra Geom.*, 4(2), 255–296

[8] Carolan, J., Harrold, C., Sparrow, C., Martín-López, E., Russell, N. J., Silverstone, J. W., Shadbolt, P. J. et al. (2015) Universal linear optics. *Science*, 349(6249), 711–716

[9] Davidson, K. R., Dor-On, A., Shalit, O. M. and Solel, B. (2017) Dilations, inclusions of matrix convex sets, and completely positive maps. *Int. Math. Res. Not. IMRN*, 13, 4069–4130

[10] Davidson, K. R., Hartz, M. and Shalit, O. M. (2015) Multipliers of embedded discs. *Complex Anal. Oper. Theory*, 9(2), 287–321

[11] Davidson, K. R. and Passer, B. (2022) Strongly peaking representations and compressions of operator systems. *Int. Math. Res. Not. IMRN*, 2022(7), 5037–5070

[12] Davidson, K. R., Ramsey, C. and Shalit, O. M. (2011) The isomorphism problem for some universal operator algebras. *Adv. Math.*, 228(1), 167–218

[13] Davidson, K. R., Ramsey, C. and Shalit, O. M. (2015) Operator algebras for analytic varieties. *Trans. Amer. Math. Soc.*, 367(2), 1121–1150

[14] Delgado-Granados, L. H. et al. (2025) Quantum algorithms and applications for open quantum systems. *Chemical Reviews*, 125(4), 1823–1839

[15] Fritz, T., Netzer, T. and Thom, A. (2017) Spectrahedral containment and operator systems with finite-dimensional realization. *SIAM J. Appl. Algebra Geom.*, 1(1), 556–574

[16] Gerhold, M., Pandey, S. K., Shalit, O. M. and Solel, B. (2021) Dilations of unitary tuples. *J. Lond. Math. Soc. (2)*, 104(5), 2053–2081

[17] Gerhold, M., Scherer, M. and Shalit, O. M. (2025) Empirical bounds for commuting dilations of free unitaries and the universal dilation constant. *arXiv*, 2510.12540

[18] Gerhold, M. and Shalit, O. (to appear) Dilation distance and the stability of ergodic commutation relations. *under revision for J. Funct. Anal.*

[19] Gerhold, M. and Shalit, O. M. (2021) On the matrix range of random matrices. *J. Operator Theory*, 85(2), 527–545

[20] Gerhold, M. and Shalit, O. M. (2022) Dilations of q-commuting unitaries. *Int. Math. Res. Not. IMRN*, 2022(1), 63–88

[21] Gerhold, M. and Shalit, O. M. (2023) Bounded perturbations of the Heisenberg commutation relation via dilation theory. *Proc. Amer. Math. Soc.*, 151(9), 3949–3957

[22] Hartz, M. (2012) Topological isomorphisms for some universal operator algebras. *J. Funct. Anal.*, 263(11), 3564–3587

[23] Hartz, M. and Lupini, M. (2021) Dilation theory in finite dimensions and matrix convexity. *Israel J. Math.*, 245(1), 39–73

[24] Hartz, M., Richter, S. and Shalit, O. M. (2022) von Neumann's inequality for row contractive matrix tuples. *Math. Z.*, 301(4), 3877–3894

[25] Head-Marsden, K. et al. (2020) Quantum information and algorithms for correlated quantum matter. *Chemical Reviews*, 121(5), 3061–3120

[26] Helton, J. W., Klep, I., McCullough, S. and Schweighofer, M. (2019) Dilations, linear matrix inequalities, the matrix cube problem and beta distributions. *Mem. Amer. Math. Soc.*, 257(1232), vi+106

[27] Jury, M. T. and Martin, R. T. W. (2020) Column extreme multipliers of the free Hardy space. *J. Lond. Math. Soc. (2)*, 101(2), 457–489

[28] Jury, M. T. and Martin, R. T. W. (2022) Fatou's theorem for non-commutative measures. *Adv. Math.*, 400, Paper No. 108293, 53

[29] Kennedy, M. (2015) Essential normality and the decomposability of homogeneous submodules. *Trans. Amer. Math. Soc.*, 367(1), 293–311

[30] Kennedy, M. and Shalit, O. M. (2012) Essential normality and the decomposability of algebraic varieties. *New York J. Math.*, 18, 877–890

[31] Kennedy, M. and Shalit, O. M. (2015) Essential normality, essential norms and hyperrigidity. *J. Funct. Anal.*, 268(10), 2990–3016

[32] Kerr, M., McCarthy, J. E. and Shalit, O. M. (2013) On the isomorphism question for complete Pick multiplier algebras. *Integral Equations Operator Theory*, 76(1), 39–53

[33] Levy, E. and Shalit, O. M. (2009) Continuous extnesion of a densely parametrized semigroup. *Semigroup Forum*, 78, 276–284

[34] Markiewicz, D. and Shalit, O. M. (2010) Continuity of CP-semigroups in the point-strong operator topology. *J. Operator Theory*, 64, 149–154

[35] McCarthy, J. and Shalit, O. M. (2013) Unitary N-dilations for tuples of commuting matrices. *Proc. Amer. Math. Soc.*, 141(2), 563–571

[36] Passer, B., Shalit, O. M. and Solel, B. (2018) Minimal and maximal matrix convex sets. *J. Funct. Anal.*, 274(11), 3197–3253

[37] Salomon, G., Shalit, O. M. and Shamovich, E. (2018) Algebras of bounded noncommutative analytic functions on subvarieties of the noncommutative unit ball. *Trans. Amer. Math. Soc.*, 370(12), 8639–8690

[38] Salomon, G., Shalit, O. M. and Shamovich, E. (2020) Algebras of noncommutative functions on subvarieties of the noncommutative ball: the bounded and completely bounded isomorphism problem. *J. Funct. Anal.*, 278(7), 108427, 54

[39] Sampat, J. and Shalit, O. M. (2025) On the classification of function algebras on subvarieties of noncommutative operator balls. *J. Funct. Anal.*, 288(2), Paper No. 110703

[40] Sampat, J. and Shalit, O. M. (2025) Weak-* and completely isometric structure of noncommutative function algebras. *J.Math. Anal. Appl.*, 550(1), 129552

[41] Shalit, O. M. (2008) E_0-dilation of strongly commuting CP_0-semigroups. *J. Funct. Anal.*, 255, 46–89

[42] Shalit, O. M. (2011) E-dilation of strongly commuting CP-semigroups (the nonunital case). *Houston J. Math.*, 35, 203–232

[43] Shalit, O. M. (2011) Stable polynomial division and essential normality of graded Hilbert modules. *J. Lond. Math. Soc. (2)*, 83(2), 273–289

[44] Shalit, O. M. and Shamovich, E. (2025) A spectral radius for matrices over an operator space. *Adv. Math.*, 479, 110449

[45] Shalit, O. M. and Skeide, M. (2023) CP-semigroups and dilations, subproduct systems and superproduct systems: the multi-parameter case and beyond. *Dissertationes Mathematicae*, 585, 1–233

[46] Shalit, O. M. and Solel, B. (2009) Subproduct systems. *Doc. Math.*, **14**, 801–868