On the spectral radius associated with an operator space

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A similarity problem

Problem

Given d-tuple of operators $A = (A_1, \ldots, A_d) \in B(\mathcal{H})^d$, does there exist an invertible operator S such that

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Tuples of commuting operators

A model: Let $\Omega \subset \mathbb{C}^d$ be a bounded domain.
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How to approach similarity problems for general tuples of operators?

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Example: $\mathcal{E} = \min(\ell_d^\infty)$ and the NC polydisc

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“The" joint spectral radius

Definition (Bunce 1984)

The **joint spectral radius** of $A = (A_1, \ldots, A_d)$ is defined to be

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Theorem (Bunce 1984/Popescu 2014)

$A \in B(\mathcal{H})^d$ is similar to a strict row contraction if and only if $\rho(A) < 1$.

About the proof: One can use a noncommutative shift on a model space and Rota's idea generalizes.

The new joint spectral radius

Definition (S-Shamovich)

Given $Q_1, \dots, Q_d \in B(\mathcal{H})$ and $A \in M_n^d$

$$\rho_Q(A) = \lim_n \| \sum_{|w|=n} A^{w_1} \cdots A^{w_n} \otimes Q_{w_1} \otimes_h \cdots \otimes_h Q_{w_n} \|^{1/n}$$

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About the proof:

1. Using model theory does not work.
2. It uses NC function theory and the theory of CB maps.

Commuting tuples revisited

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Question: Does $\rho_Q(A)$ depend on $\mathcal{E}$ in general?

Dependence on operator space structure

Definition

An operator space structure $\mathcal{E}$ on $\mathbb{C}^d$ is said to be selfadjoint if

$$\|(A_1^*, \dots, A_d^*)\| = \|(A_1, \dots, A_d)\|$$

for all $A = (A_1, \dots, A_d) \in \mathbb{M}^d$.

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For every $A \in \mathbb{M}^d$

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On the other hand, there exists $T \in B(\mathcal{H})^d$ such that $\rho_{\mathrm{row}}(T) \neq \rho_{\mathrm{col}}(T)$.

Theorem (Scherer–S–Shamovich)

Let X be a Banach space with $\dim X = d \geq 3$. Then there exist $n \in \mathbb{N}$ and $A \in M_n(X)$ such that

$$\rho_{\min(X)}(A) < \rho_{\max(X)}(A).$$

Dependence on operator space structure (continued)

Example

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Question

Let $\mathcal{E}_1$ and $\mathcal{E}_2$ be two operator space structures on $\mathbb{C}^d$. Does $\rho_{\mathcal{E}_1}(A) = \rho_{\mathcal{E}_2}(A)$ for every d-tuple of operators A imply that $\mathcal{E}_1 = \mathcal{E}_2$?

Thank you